

С2 «ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ»

Задача 1 (1 балл)

Найдите общее решение дифференциального уравнения.

- $4x dx - 3y dy = 3x^2 y dy - 2xy^2 dx.$
- $x\sqrt{1+y^2} + yy'\sqrt{1+x^2} = 0.$
- $\sqrt{4+y^2} dx - y dy = x^2 y dy.$
- $\sqrt{3+y^2} dx - y dy = x^2 y dy.$
- $4x dx - 3y dy = 3x^2 y dy - 3xy^2 dx.$
- $x\sqrt{3+y^2} dx + y\sqrt{2+x^2} dy = 0.$
- $\sqrt{3+y^2} dx + y\sqrt{1-x^2} dy = 0.$
- $x\sqrt{3+y^2} dx - y\sqrt{4+x^2} dy = 0.$
- $x\sqrt{1+y^2} dx + y\sqrt{1+x^2} dy = 0.$
- $x\sqrt{3+y^2} dx + y\sqrt{4+x^2} dy = 0.$
- $y(4+e^x) dy - e^x dx = 0.$
- $\sqrt{4-x^2} y' + xy^2 + x = 0.$
- $\sqrt{3+y^2} + \sqrt{1-x^2} yy' = 0.$
- $x dx - y dy = yx^2 dy - xy^2 dx.$
- $2x dx - 2y dy = x^2 y dy - 2xy^2 dx.$
- $x\sqrt{4+y^2} dx + y\sqrt{1+x^2} dy = 0.$
- $(e^x + 8) dy - ye^x dx = 0.$
- $\sqrt{5+y^2} + y'y\sqrt{1-x^2} = 0.$
- $(1+e^x) yy' = e^x.$
- $y \ln y + xy' = 0.$
- $(1+e^x) y' = ye^x.$
- $\sqrt{1-x^2} y' + xy^2 + x = 0.$
- $6x dx - 2y dy = 2yx^2 dy - 3xy^2 dx.$
- $x\sqrt{4+y^2} dx - y\sqrt{x^2+1} dy = 0.$
- $x\sqrt{3+y^2} dx + y\sqrt{2+x^2} dy = 0.$

Задача 2 (3 балла)

Найдите решение задачи Коши.

- $y' - \frac{y}{x} = x^2, \quad y(1) = 0.$
- $y' - \frac{y}{x} = x^2., \quad y(1) = 0.$
- $y' - \frac{y}{x} = x^4 - 1., \quad y(1) = \frac{1}{4}.$
- $y' - \frac{y}{x} = x., \quad y(1) = 1.$
- $y' + \frac{y}{x+2} = x^2 + 2x, \quad y(-1) = \frac{3}{2}.$
- $y' - \frac{y}{x} = x \sin x, \quad y\left(\frac{\pi}{2}\right) = 1.$
- $y' - \frac{y}{x+1} = e^x(x+1), \quad y(0) = 1.$
- $y' + \frac{y}{x} = x^2., \quad y(1) = \frac{1}{4}.$
- $y' + \frac{y}{2x} = x^2, \quad y(1) = 1.$
- $y' + \frac{y}{x} = x^2 - 2., \quad y(1) = 2.$
- $y' - \frac{2y}{x} = 5, \quad y(2) = 4.$
- $y' + \frac{y}{x} = \frac{e^x}{x}, \quad y(1) = e.$
- $y' + \frac{y}{x} = x^3., \quad y(1) = \frac{1}{5}.$
- $y' - \frac{y}{x} = -\frac{8}{x^2}, \quad y(1) = 4.$

15. $y' + \frac{2}{x}y = x^3, \quad y(1) = -\frac{5}{6}.$

16. $y' + \frac{y}{x} = 3x, \quad y(1) = 1.$

17. $y' - \frac{y}{x} = x^2 + 1, \quad y(1) = \frac{1}{2}.$

18. $y' + \frac{y}{x} = x^2 + 1, \quad y(1) = 1.$

19. $y' + \frac{3y}{x} = \frac{2}{x}, \quad y(1) = 1.$

20. $y' + 2xy = -2x, \quad y(0) = 1.$

21. $y' - \frac{y}{x} = x^3 + 1, \quad y(1) = \frac{1}{3}.$

22. $y' + \frac{2y}{x} = x^3, \quad y(1) = \frac{1}{6}.$

23. $y' - \frac{2y}{x+1} = (x+1)^3, \quad y(0) = \frac{1}{2}.$

24. $y' + xy = -x, \quad y(0) = 3.$

25. $y' + \frac{y}{x} = x, \quad y(1) = \frac{1}{3}.$

Задача 3 (2 балла)

Найдите общий интеграл дифференциального уравнения.

1. $3y' = \frac{y^2}{x^2} + 10\frac{y}{x} + 10.$

2. $xy' = \frac{3y^3 + 2yx^2}{2y^2 + x^2}.$

3. $y' = \frac{x+y}{x-y}.$

4. $xy' = \sqrt{x^2 + y^2} + y.$

5. $2y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 3.$

6. $xy' = \frac{3y^3 + 4yx^2}{2y^2 + 2x^2}.$

7. $y' = \frac{x+2y}{2x-y}.$

8. $xy' = 2\sqrt{x^2 + y^2} + y.$

9. $3y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 4.$

10. $xy' = \frac{3y^3 + 6yx^2}{2y^2 + 3x^2}.$

11. $y' = \frac{x^2 + xy - y^2}{x^2 - 2xy}.$

12. $xy' = \sqrt{2x^2 + y^2} + y.$

13. $y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 6.$

14. $xy' = \frac{3y^3 + 8yx^2}{2y^2 + 4x^2}.$

15. $y' = \frac{x^2 + 2xy - y^2}{2x^2 - 2xy}.$

16. $xy' = 3\sqrt{x^2 + y^2} + y.$

17. $2y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 8.$

18. $xy' = \frac{3y^3 + 10yx^2}{2y^2 + 5x^2}.$

19. $y' = \frac{x^2 + 3xy - y^2}{3x^2 - 2xy}.$

20. $xy' = 3\sqrt{2x^2 + y^2} + y.$

21. $y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 12.$

22. $xy' = \frac{3y^3 + 12yx^2}{2y^2 + 6x^2}.$

$$23. y' = \frac{x^2 + xy - 3y^2}{x^2 - 4xy}.$$

$$24. xy' = 2\sqrt{3x^2 + y^2} + y.$$

$$25. 4y' = \frac{y^2}{x^2} + 10\frac{y}{x} + 5.$$

Задача 4 (4 балла)

Найдите решение задачи Коши.

1. $y'' - 3y' + 2y = e^{-x}$, $y(0) = 1$, $y'(0) = 1$.
2. $y'' - y' - 2y = 2x + 2$, $y(0) = 0$, $y'(0) = 1$.
3. $y'' - y' - 2y = -2x - 1$, $y(0) = 0$, $y'(0) = 2$.
4. $y'' + 2y' + y = xe^x$, $y(0) = 2$, $y'(0) = 0$.
5. $2y'' + 8y = 5x^2 - 2x - 1$, $y(0) = 0$, $y'(0) = 0$.
6. $y'' + y' - 2y = \sin x - 2\cos x$, $y(0) = 0$, $y'(0) = 0$.
7. $y'' + 2y' + y = 6xe^x$, $y(0) = 1$, $y'(0) = 0$.
8. $y'' - y = (2x + 1)e^{2x}$, $y(0) = 0$, $y'(0) = 1$.
9. $y'' - 6y' + 9y = 2x^2 - x + 3$, $y(0) = 0$, $y'(0) = 1$.
10. $y'' + 2y' + y = (3x + 2)e^x$, $y(0) = 1$, $y'(0) = 1$.
11. $y'' + y' - 2y = \cos x - 3\sin x$, $y(0) = 1$, $y'(0) = 1$.
12. $y'' - 3y' + 2y = \cos x - \sin x$, $y(0) = 1$, $y'(0) = 1$.
13. $y'' - 9y = (4x + 2)e^x$, $y(0) = 1$, $y'(0) = 0$.
14. $y'' - y = 5x$, $y(0) = 0$, $y'(0) = 1$.
15. $y'' + 2y' + y = (9x + 6)e^{2x}$, $y(0) = 1$, $y'(0) = 1$.
16. $y'' - y = e^{2x}$, $y(0) = 0$, $y'(0) = 2$.
17. $y'' - 3y' + 2y = \sin x - 3\cos x$, $y(0) = 1$, $y'(0) = 1$.
18. $y'' - 2y' - 3y = \cos x + 2\sin x$, $y(0) = 2$, $y'(0) = 0$.
19. $y'' - 4y = 2e^x$, $y(0) = 1$, $y'(0) = 2$.
20. $y'' - y = 3e^{-x}$, $y(0) = 0$, $y'(0) = 2$.
21. $y'' - 4y = 4e^{2x}$, $y(0) = 0$, $y'(0) = 2$.
22. $y'' - y' = 1 + 2x$, $y(0) = 0$, $y'(0) = 2$.
23. $y'' - 2y' - 3y = 2\sin x - 4\cos x$, $y(0) = 2$, $y'(0) = 0$.
24. $y'' - 3y' + 2y = \cos x - \sin x$, $y(0) = 0$, $y'(0) = 0$.
25. $y'' - y = -2e^{-x}$, $y(0) = 0$, $y'(0) = 2$.