

КОНТРОЛЬНАЯ ТОЧКА СЗ (МА, 1сем, тех, 2020)

Каждая задача 1 балл.

Задачи 1,2,3

Вычислите производные функций.

- | | | |
|--|---|---|
| 1.1 $y = x^3 \ln 3x$; | 2.1 $y = \sqrt{x^3} \operatorname{ctg}(x^2)$; | 3.1 $y = e^{-2x} \operatorname{arctg} 3x$. |
| 1.2 $y = 2x^3 \operatorname{ctg} x$; | 2.2 $y = \frac{\ln \cos 2x}{3x^2 + 1}$; | 3.2 $y = e^{-x^2} \sin 5x^3$. |
| 1.3 $y = (1 + x^2) \operatorname{tg} 3x$; | 2.3 $y = \frac{\arccos x^2}{1 - x^4}$; | 3.3 $y = \operatorname{tg}^2(3x^3) - x$. |
| 1.4 $y = x^3 \sin 5x$; | 2.4 $y = 5x^2 \sqrt{1 - 2x^3}$; | 3.4 $y = \operatorname{arctg}(\ln 5x^2)$. |
| 1.5 $y = e^x(x^3 - 2x + 1)$; | 2.5 $y = \cos\left(\frac{\pi}{6} - 3x^2\right)$; | 3.5 $y = x^3 \cdot 2^{-\cos 5x}$. |
| 1.6 $y = \frac{\operatorname{tg} x \ln x}{5^x}$; | 2.6 $y = 3^{2x} \operatorname{ctg} 2x^3$; | 3.6 $y = e^{-2x} \ln \operatorname{tg} 3x$. |
| 1.7 $y = 6^x \arccos x$; | 2.7 $y = \sqrt[3]{2 \operatorname{tg} 3x}$; | 3.7 $y = \sin^3 2x \cdot e^{-\cos 5x}$. |
| 1.8 $y = 2x^3 \log_4 x$; | 2.8 $y = e^{-x} \sqrt{3x^2 - 4x + 5}$; | 3.8 $y = \operatorname{arctg} \frac{2x}{1 - x^2}$. |
| 1.9 $y = 6^x \cos 3x$; | 2.9 $y = \sqrt[3]{3 \operatorname{tg}^2 5x}$; | 3.9 $y = \operatorname{arctg}(\ln 5x^2)$. |
| 1.10 $y = 2x^3 \operatorname{tg} x$; | 2.10 $y = e^{-\sin 3x} \ln 5x^2$; | 3.10 $y = 2 \operatorname{tg}^3(3x^2 - x - 1)$. |
| 1.11 $y = \frac{\operatorname{ctg} x}{2 + \sin x}$; | 2.11 $y = 3 \cos(2x^2 - x - 1)$; | 3.11 $y = e^{-2x} \ln(\sin 3x)$. |
| 1.12 $y = \frac{3 \cos x}{2x^3 + 1}$; | 2.12 $y = 2^{-x^3} \sin^3 x^2$; | 3.12 $y = \sqrt[3]{x^2 - 6\sqrt{x}}$. |
| 1.13 $y = 3x^2 \sin 2x$; | 2.13 $y = \cos^2 8x \ln x$; | 3.13 $y = \cos 2^x + 4^{-x^3}$. |
| 1.14 $y = (\sin x + \cos x) \sqrt[3]{x^{21}}$; | 2.14 $y = 2^4 \sqrt{\arcsin 2x}$; | 3.14 $y = 3^{2x^2} \operatorname{tg}(\ln 3x)$. |
| 1.15 $y = (2x + 1)^{11} \cdot \sqrt[3]{x^2}$; | 2.15 $y = \cos(3x^2 - 2x + 1)$; | 3.15 $y = 2 \operatorname{tg}^3(3x^2 - 1)$. |
| 1.16 $y = \frac{-5 \sin x}{2 - \sqrt{x}}$; | 2.16 $y = 3x^2 \cos^2(x^3 - 1)$; | 3.16 $y = 3\sqrt{x^3} \ln(2x^3 - 1)$. |
| 1.17 $y = \frac{x^3 - \sin x}{\sqrt{x^3}}$; | 2.17 $y = \sqrt[4]{\arccos 3x^3}$; | 3.17 $y = \ln(e^{3x} + xe^{-x^3})$. |

1.18 $y = \cos(1 - \pi x) + \sin 3x;$	2.18 $y = \operatorname{arctg} \sqrt[3]{x^2};$	3.18 $y = 10^{5-3x^2}.$
1.19 $y = 2x^3 \log_4 x;$	2.19 $y = 2e^{-3x} \operatorname{tg} 5x^2;$	3.19 $y = \frac{\arcsin 2x^3}{\sqrt[3]{x^2}}.$
1.20 $y = \frac{1 + 4 \sin x}{2 - 3 \cos x};$	2.20 $y = \frac{\operatorname{ctg} 2x^3}{2^{3-2x^2}};$	3.20 $y = 3 \ln(x^2 + \sqrt{1+x}).$
1.21 $y = -8 \sqrt[4]{x^3} \arcsin x;$	2.21 $y = x^3 \operatorname{tg}^2 3x^4;$	3.21 $y = e^{-\cos 2x} \sqrt{\sin 3x}.$
1.22 $y = \frac{\log_2 x + 1}{\sqrt[5]{x^2}};$	2.22 $y = (3 - 2x^3) \sin^2 3x;$	3.22 $y = \arccos(1 - x^2).$
1.23 $y = -3 \sqrt[5]{3x} \operatorname{arctg} x;$	2.23 $y = 2 \sin^6 \left(1 - \sqrt[3]{x^2}\right);$	3.23 $y = \sin 2x \cdot e^{\operatorname{ctg}^2 x}.$
1.24 $y = 4^x \arccos x - \frac{e^x}{x^2};$	2.24 $y = \frac{\sqrt[3]{x^2 - 6x}}{x^{12}};$	3.24 $y = \frac{1}{3} \sin^3 2x e^{-\cos 2x}.$
1.25 $y = -3 \arcsin x + 4 \sqrt{2x};$	2.25 $y = \frac{1}{6} x^6 (e^{6x} - e^{-6x});$	3.25 $y = \frac{1}{2} x \ln(e^{-2x} + x e^{-2x}).$
1.26 $y = \frac{\ln x - 1}{\sqrt[3]{x^2 + 1}};$	2.26 $y = \sqrt[5]{\arccos x^3};$	3.26 $y = \ln(x + \sqrt{2 + x^2}).$
1.27 $y = 3^x \operatorname{arctg} x - \sqrt{1 - x^2};$	2.27 $y = x^4 (e^{4x} - e^{-4x});$	3.27 $y = \log_2(e^x + x e^x).$
1.28 $y = 3 \cos \sqrt{\pi x} + \sin^2 3x;$	2.28 $y = \frac{x^2}{\sqrt[3]{x^2 + 3x}};$	3.28 $y = x \cdot 7^{5-2x+3x^2}.$
1.29 $y = \arcsin x^2 + \sqrt{x^2 + 3};$	2.29 $y = x^3 \operatorname{tg} \frac{x^4}{3};$	3.29 $y = \sqrt{x} \operatorname{ctg} 5x^3;$
1.30 $y = e^x \ln(x^3 - 2);$	2.30 $y = \ln(x + \cos^2 8x);$	3.30 $y = \frac{\log_2(x-1)}{\sqrt[4]{3x^2 + 4}}.$

Задача 4

Вычислите предел, используя правило Лопиталья.

4.1 $\lim_{x \rightarrow 0} \frac{\ln \operatorname{tg} x}{\ln \sin x}.$	4.4 $\lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 2 \sin x}{\cos 3x}.$	4.7 $\lim_{x \rightarrow 0} \frac{e^{3x} - 3x - 1}{\sin^2 x}.$
4.2 $\lim_{x \rightarrow 0} \frac{\ln \sin 2x}{\ln \operatorname{tg} x}.$	4.5 $\lim_{x \rightarrow 0} \frac{\ln x}{\operatorname{ctg} x}.$	4.8 $\lim_{x \rightarrow 2} \frac{\ln(x-1)}{\sqrt{x-2}}.$
4.3 $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \operatorname{tg} x}{0,5 - \sin^2 x}.$	4.6 $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{ctg} x - 1}{\sin 4x}.$	4.9 $\lim_{x \rightarrow 2\pi} \frac{\ln \cos x}{\sin x}.$

$$\begin{array}{lll}
4.10 \quad \lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{tg} 3x}{\operatorname{tg} 5x} & 4.18 \quad \lim_{x \rightarrow 5} \frac{\sqrt{5} - \sqrt{x}}{\sqrt[3]{5} - \sqrt[3]{x}} & 4.26 \quad \lim_{x \rightarrow +\infty} \frac{x^2}{e^{4x}} \\
4.11 \quad \lim_{x \rightarrow 0} \frac{x - \operatorname{arctg} x}{x^3} & 4.19 \quad \lim_{x \rightarrow 0} \frac{1 - 2^x}{1 - 3^x} & 4.27 \quad \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin 6x}{\cos 3x} \\
4.12 \quad \lim_{x \rightarrow 0} \frac{2^x - 3^x}{\operatorname{tg} 2x} & 4.20 \quad \lim_{x \rightarrow 1} \frac{2 - 2^x}{\ln x} & 4.28 \quad \lim_{x \rightarrow \frac{\pi}{3}} \frac{x - \frac{\pi}{3} - \sin 3x}{\cos \frac{3x}{2}} \\
4.13 \quad \lim_{x \rightarrow 3} \frac{\sqrt[5]{x} - \sqrt[5]{3}}{\log_2(x-2)} & 4.21 \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \operatorname{tg} x}{\cos 2x} & 4.29 \quad \lim_{x \rightarrow \frac{\pi}{6}} \frac{x - \frac{\pi}{6} - \cos 3x}{\sin 6x} \\
4.14 \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{x - \sin x} & 4.22 \quad \lim_{x \rightarrow 0} \frac{e^{3x} - \cos 3x}{e^{2x} - \cos x} & 4.30 \quad \lim_{x \rightarrow 1} \frac{3^x - 3}{\ln x} \\
4.15 \quad \lim_{x \rightarrow 0} \frac{\ln \operatorname{tg} 7x}{\ln \operatorname{tg} 2x} & 4.23 \quad \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} & \\
4.16 \quad \lim_{x \rightarrow 0} \frac{x - \operatorname{arctg} x}{x^3} ; & 4.24 \quad \lim_{x \rightarrow 0} \frac{\sin x - \operatorname{tg} x}{x - \sin x} & \\
4.17 \quad \lim_{x \rightarrow 0} \frac{\ln \sin 3x}{\ln \sin x} & 4.25 \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{tg} x - 1}{\sin 4x} &
\end{array}$$

Задача 5

Найдите наибольшее и наименьшее значения функции $y = f(x)$ на отрезке $[a; b]$.

$$\begin{array}{ll}
5.1 \quad f(x) = \frac{1}{x^2 - 1}, \quad \left[-\frac{1}{2}; \frac{1}{2}\right]. & 5.11 \quad f(x) = 2x^3 + 3x^2 + 6, \quad [-3; 2]. \\
5.2 \quad f(x) = \frac{6x - x^2}{4x + 1}, \quad [0; 2]. & 5.12 \quad f(x) = \frac{x}{x^2 + 1}, \quad [-3; 3]. \\
5.3 \quad f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x - 7, \quad [1; 5]. & 5.13 \quad f(x) = \frac{6x}{x^2 + 9}, \quad [-4; 4]. \\
5.4 \quad f(x) = \frac{1}{3}x^3 - 2x^2 + 3, \quad [-1; 2]. & 5.14 \quad f(x) = \frac{8x}{x^2 + 4}, \quad [-3; 4]. \\
5.5 \quad f(x) = x^4 - 8x^2 + 3, \quad [-2; 2]. & 5.15 \quad f(x) = 2\sqrt{x} - x, \quad [0; 4]. \\
5.6 \quad f(x) = \frac{1}{3}x^3 - \frac{7}{2}x^2 + 12x - 1, \quad [1; 5]. & 5.16 \quad f(x) = x^4 - 2x^2 + 4, \quad [-2; 2]. \\
5.7 \quad f(x) = x^3 - 9x^2 + 15x - 3, \quad [0; 6]. & 5.17 \quad f(x) = \frac{x}{5} + \frac{5}{x}, \quad [1; 6]. \\
5.8 \quad f(x) = \frac{1}{4}x^4 - 2x^2 + 5, \quad [-3; 4]. & 5.18 \quad f(x) = \frac{1}{3}x^3 - 2x^2 + 3x - 1, \quad [0; 4]. \\
5.9 \quad f(x) = -2x^3 - 3x^2 + 3, \quad [-3; 5]. & 5.19 \quad f(x) = \sqrt[3]{x} - \frac{4}{3}x, \quad [1; 8]. \\
5.10 \quad f(x) = x^3 - 3x + 2, \quad [-2; 3]. &
\end{array}$$

5.20 $f(x) = \frac{x^3}{3} + x^2, [-3; 1].$

5.21 $f(x) = x^3 + 6x^2 + 9x, [-4; 1].$

5.22 $f(x) = x \ln x, [1; e].$

5.23 $f(x) = \frac{x^4}{4} - 2x^2, [-3; 3].$

5.24 $f(x) = 2x^3 - 3x^2, [-2; 2].$

5.25 $f(x) = \frac{x^2 - 1}{x^2 - 3}, \left[-\frac{1}{2}; \frac{1}{2}\right].$

5.26 $f(x) = (x+1)(x-2)^2, [2; 3].$

5.27 $f(x) = \frac{6x - x^2}{4x + 8}, [0; 1].$

5.28 $f(x) = \frac{x^3}{x^2 - 1}, [2; 3].$

5.29 $f(x) = \frac{x^2}{x - 2}, [0; 1].$

5.30 $f(x) = \frac{1}{x^2 - 4}, [3; 4].$

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Задача 6

Выполните полное исследование и постройте график функции (3 балла).

$$6.1 \quad y = 2x - \frac{x^2}{2} - \frac{x^3}{3}.$$

$$6.2 \quad y = x^4 - 2x^2 + 4.$$

$$6.3 \quad y = 2x^3 + 3x^2 + 6.$$

$$6.4 \quad y = -2x^3 - 3x^2 + 3.$$

$$6.5 \quad y = x^3 - 3x^2 + 3.$$

$$6.6 \quad y = \frac{1}{4}x^4 - 2x^2 + 5.$$

$$6.7 \quad y = x^3 - 3x + 1.$$

$$6.8 \quad y = x^3 + 6x^2 + 9x.$$

$$6.9 \quad y = \frac{x^3}{3} + x^2.$$

$$6.10 \quad y = \frac{x^4}{4} - 2x^2.$$

$$6.11 \quad y = 3x^4 - 8x^3 + 6x^2.$$

$$6.12 \quad y = \frac{x^3}{3} - x^2 - 3x.$$

$$6.13 \quad y = 1 - 2x^2 - \frac{x^4}{4}.$$

$$6.14 \quad y = \frac{x^4}{4} - x^3.$$

$$6.15 \quad y = x^3 + \frac{x^4}{4}.$$

$$6.16 \quad y = 3x^5 - 5x^3.$$

$$6.17 \quad y = x^3 - x^2.$$

$$6.18 \quad y = x^3 - 3x^2 + 4.$$

$$6.19 \quad y = \frac{x^4}{4} + \frac{x^3}{3} - x^2 - 12.$$

$$6.20 \quad y = (x+1)(x-2)^2.$$

$$6.21 \quad y = 7 + 2x + \frac{1}{2}x^2 - \frac{2}{3}x^3 - \frac{1}{4}x^4.$$

$$6.22 \quad y = x^2(1-x).$$

$$6.23 \quad y = \frac{x^3}{12} - \frac{x^2}{2}.$$

$$6.24 \quad y = -x^4 + 2x^2 + 8.$$

$$6.25 \quad y = x(x-1)^2.$$

$$6.26 \quad y = x(x^2-1).$$

$$6.27 \quad y = x^4 - 6x^2.$$

$$6.28 \quad y = x^3 + 3x^2.$$

$$6.29 \quad y = \frac{1}{3}x^3 - x^2 - 3x.$$

$$6.30 \quad y = \frac{3}{4}x^4 - x^3 - 9x^2 + 7$$

Задача 7.

Вычислите частные производные первого порядка функции (2 балла).

1 $z = 2x^2 + xy + y^2 + 2x - 7y + 1$	16 $z = 3x + 6y - x^2 - xy - y^2$
2 $z = x^2 + xy + y^2 + x - y + 1$	17 $z = 3x - 7y - 8x^2y^3 + 7y^5x^3$
3 $z = x^3 - 3xy + y^3$	18 $z = 3 - 2x^2 - xy - y^2$
4 $z = 2x^2 - xy + y^2 + 2y$	19 $z = x^2 + xy + 4y^2 - 2x$
5 $z = \cos x + 3x^2y - 4y^3x$	20 $z = x^2 - 2xy + 2y^2 + 2x$
6 $z = 2x^2 + xy + y^2 + 2x - 7y + 1$	21 $z = x^2 - xy + y^2 - 1$
7 $z = (x - 3y)^2 - 2(x + y) - 2$	22 $z = 3x^4y - 5xy^3 + y - x - xy$
8 $z = -2x^2 + xy + 4y^2 - x + 5y + 4$	23 $z = 8x^3 - 12xy - y^3 - 1$
9 $z = -x^2 + 4xy - 3y^2 + 5x - 3y + 2$	24 $z = 2x^3 - 12x^2y - 9x^2 + 16y^3$
10 $z = x^2 + 5xy - 4y^2 + x - 7y + 5$	25 $z = x^2y - y^3 - x^2 - 3y^2 + 3$
11 $z = (2x - y)^2 + x^2 - 4y + 3$	26 $z = 8x^3y - 3xy + 2$
12 $z = 2x^2 + xy + y^2 - \frac{(x + y)}{2}$	27 $z = \frac{x^2}{4} + \frac{y^2}{2} - \frac{xy}{2} + y - 2x + 2$
13 $z = \frac{x^2}{2} + 2xy - 4y^2 + 5x + 6y + 3$	28 $z = \frac{x^2}{4} + \frac{y^2}{2} - 2xy + x - 4y + 8$
14 $z = x^2 + 2xy - 4x + 4y - y^2 + 5$	29 $z = 4xy + 2y^2 - 3x^2 + 4x - 7y + 4$
15 $z = x^2 + 2xy - 4x + 4y - y^3$	30 $z = 2x^2 + xy + y^2 + 2x - 7y + 1$