

Контрольная точка С1 (АТ, 4 семестр, 2016)

Задача 1 (а, б – по 1 баллу, в – 2 балла)

Исследуйте ряды на сходимость.

- | | | |
|--|--|---|
| 1. а) $\sum_{n=1}^{\infty} \frac{1}{2n-1};$ | б) $\sum_{n=1}^{\infty} \frac{7^{2n}}{(2n-1)!};$ | в) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{2n+1}{3n+1} \right)^n.$ |
| 2. а) $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{\sqrt{n}};$ | б) $\sum_{n=1}^{\infty} \frac{n^2}{(n+2)!};$ | в) $\sum_{n=1}^{\infty} (-1)^n \sin \frac{\pi}{2^n}.$ |
| 3. а) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{(n+3)^3};$ | б) $\sum_{n=1}^{\infty} \left(\frac{n-1}{3n} \right)^n;$ | в) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2n+1}{n^2+2} \right)^2.$ |
| 4. а) $\sum_{n=1}^{\infty} \frac{(n+1)^2}{n \cdot 3^{n-1}};$ | б) $\sum_{n=1}^{\infty} \left(\frac{2n+3}{n+1} \right)^{n^2};$ | в) $\sum_{n=1}^{\infty} (-1)^n \frac{3n+2}{4n^2+1}.$ |
| 5. а) $\sum_{n=1}^{\infty} \frac{\arctg n}{3n^2+1};$ | б) $\sum_{n=1}^{\infty} \frac{10^n \cdot n}{3^n};$ | в) $\sum_{n=1}^{\infty} (-1)^n \frac{2n+3}{(3n+2)^2}.$ |
| 6. а) $\sum_{n=1}^{\infty} \left(\frac{3n+1}{6n+2} \right)^{5n+3};$ | б) $\sum_{n=1}^{\infty} \frac{n+1}{2^n};$ | в) $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{\sqrt{4n^3+1}}.$ |
| 7. а) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n};$ | б) $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n+3} \right)^{n^2};$ | в) $\sum_{n=1}^{\infty} (-1)^n \frac{2n-1}{3n+2}.$ |
| 8. а) $\sum_{n=1}^{\infty} \frac{5n}{n^3+1};$ | б) $\sum_{n=1}^{\infty} n! \cdot \sin \frac{\pi}{2^n};$ | в) $\sum_{n=1}^{\infty} (-1)^n \frac{2n-1}{\sqrt{9n^3+1}}.$ |
| 9. а) $\sum_{n=1}^{\infty} 4 \sqrt[4]{\frac{n}{n^6+1}};$ | б) $\sum_{n=1}^{\infty} \frac{5^n \cdot \sqrt[3]{n^2}}{(n+1)!};$ | в) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{(n+1) \cdot 2^{2n}}.$ |
| 10. а) $\sum_{n=1}^{\infty} \frac{\sin^2 n^2}{\sqrt{n}(n+1)};$ | б) $\sum_{n=1}^{\infty} \left(\frac{2n}{4n+3} \right)^{2n};$ | в) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2n+3}{3n^2-2} \right)^2.$ |
| 11. а) $\sum_{n=1}^{\infty} \frac{n^2+3}{n^3(2+(-1)^n)};$ | б) $\sum_{n=1}^{\infty} \frac{1}{(2n+1) \cdot 2^{2n+1}};$ | в) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n \cdot \sqrt[4]{16n+3}}.$ |
| 12. а) $\sum_{n=1}^{\infty} \frac{2n^2+3n+1}{4n^3+5n};$ | б) $\sum_{n=1}^{\infty} \frac{(n+1)!}{2^n};$ | в) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n}{\sqrt{n^3+4}}.$ |
| 13. а) $\sum_{n=1}^{\infty} \frac{n^2+1}{\sqrt[3]{n^7+3n}};$ | б) $\sum_{n=1}^{\infty} \frac{4^{n+1}}{n^2+n-1};$ | в) $\sum_{n=1}^{\infty} \frac{(-1)^n (2n+1)}{3n^2+5}.$ |
| 14. а) $\sum_{n=1}^{\infty} \frac{n}{2n^2-1};$ | б) $\sum_{n=1}^{\infty} \frac{(n+1)^2}{2^n(2n+7)};$ | в) $\sum_{n=1}^{\infty} (-1)^n \frac{2n^2}{n^4-n^2+1}.$ |

15.a) $\sum_{n=1}^{\infty} \frac{5^n}{(n+1)!};$	б) $\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)^n};$	в) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{2n^3-1}.$
16.a) $\sum_{n=1}^{\infty} \frac{3n+7}{n+1};$	б) $\sum_{n=1}^{\infty} \frac{5^n(n+1)}{n^2+n+5};$	в) $\sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{n(n+1)}.$
17.a) $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n^3};$	б) $\sum_{n=1}^{\infty} \frac{n^2+1}{4^n};$	в) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{(\sqrt{n}+1)^5}.$
18.a) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n\sqrt{n}};$	б) $\sum_{n=1}^{\infty} \frac{3 \cdot 4^{n-1}}{n^4+1};$	в) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n^3+5}.$
19.a) $\sum_{n=1}^{\infty} \frac{1}{(5n-2)^2};$	б) $\sum_{n=1}^{\infty} \frac{1}{n \cdot 5^n};$	в) $\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{3n+1}.$
20.a) $\sum_{n=1}^{\infty} \frac{n}{n^3+3};$	б) $\sum_{n=1}^{\infty} \frac{3^n}{n!};$	в) $\sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{n(n+1)}.$
21.a) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^4+1};$	б) $\sum_{n=1}^{\infty} \frac{n^3}{2^n};$	в) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^4+n^2+1}.$
22.a) $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1};$	б) $\sum_{n=1}^{\infty} \frac{n^2+8}{n^2 \cdot 2^n};$	в) $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{\sqrt{n^3}}.$
23.a) $\sum_{n=1}^{\infty} \sqrt{\frac{n^2+3}{3n^4-1}};$	б) $\sum_{n=1}^{\infty} \frac{6^n(n^2-1)}{n!};$	в) $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{(n+1)!}.$
24.a) $\sum_{n=1}^{\infty} \frac{(n+4)2^n}{(n-1)!};$	б) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n;$	в) $\sum_{n=1}^{\infty} (-1)^n \frac{3n+4}{n(5n+1)}.$
25.a) $\sum_{n=1}^{\infty} \frac{(3n+1)(n+1)!}{7^n};$	б) $\sum_{n=1}^{\infty} \left(\frac{2n}{3n-2}\right)^n;$	в) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^3+n-1}.$

Задача 2 (2 балла)

Найдите область сходимости функционального ряда.

1. $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{(n+1) \cdot 5^n}.$	4. $\sum_{n=1}^{\infty} \frac{(x-5)^n}{(n+4) \cdot 4^n}.$	7. $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n}.$
2. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} (x-2)^n}{2n}.$	5. $\sum_{n=1}^{\infty} \frac{(x-7)^n}{n^2 \cdot 2^n}.$	8. $\sum_{n=1}^{\infty} \frac{(x+1)^n \cdot n}{(n+1)!}.$
3. $\sum_{n=1}^{\infty} (x+5)^n \cdot \operatorname{tg} \frac{1}{3^n}.$	6. $\sum_{n=1}^{\infty} \frac{(-1)^n (x+5)^n}{\sqrt{n+1}}.$	9. $\sum_{n=1}^{\infty} \frac{x^n}{n!}.$

$$\begin{array}{lll}
10. \sum_{n=1}^{\infty} \frac{4^n (x+1)^n}{n} & 16. \sum_{n=1}^{\infty} \frac{(n+1)^3 (x+3)^n}{2n+3} & 22. \sum_{n=1}^{\infty} \frac{(x-5)^n}{3n+8} \\
11. \sum_{n=1}^{\infty} \frac{(n-1)(x+3)^n}{3^{n+1}} & 17. \sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 5^n} & 23. \sum_{n=1}^{\infty} \frac{(x-7)^n}{(2n^2+5) \cdot 4^n} \\
12. \sum_{n=1}^{\infty} \frac{(x-2)^n}{(3n+1) \cdot 2^n} & 18. \sum_{n=1}^{\infty} \frac{(x+5)^n}{4^n (2n-1)} & 24. \sum_{n=1}^{\infty} (x-2)^n \cdot \sin \frac{1}{2^n} \\
13. \sum_{n=1}^{\infty} \frac{(n-1)(x+3)^n}{(n+1)!} & 19. \sum_{n=1}^{\infty} \frac{x^n}{2^{n-1}} & 25. \sum_{n=1}^{\infty} \frac{(x-1)^n \cdot n}{n^2+2n+3} \\
14. \sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{(n+1) \cdot 5^n} & 20. \sum_{n=1}^{\infty} x^n \operatorname{tg} \frac{1}{n} & \\
15. \sum_{n=1}^{\infty} \frac{n^5 (x+5)^n}{(n+1)!} & 21. \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-2)^n}{n \cdot 4^n} &
\end{array}$$

Задача 3 (2 балла)

Найдите n первых отличных от нуля членов разложения решения задачи Коши в степенной ряд.

1. $y' = x^3 + y^3$, $y(0) = 1$, $n = 3$.
2. $y'' - x^2 y' - y = 0$, $y(0) = 1$, $y'(0) = 0$, $n = 3$.
3. $y'' = e^x + y^2$, $y(0) = 0$, $y'(0) = 1$, $n = 4$.
4. $y'' = xy - 5y'$, $y(0) = 2$, $y'(0) = 1$, $n = 5$.
5. $y' = \cos 4x - x^2 - y^2$, $y(0) = 0$, $n = 2$.
6. $y' = 2x^2 - y^3 - 2xy$, $y(0) = 1$, $n = 4$.
7. $y'' - xy + \sin x = 0$, $y(0) = y'(0) = 1$, $n = 3$.
8. $y' = y^2 + \cos x$, $y(0) = 1$, $n = 4$.
9. $y'' = (3x-2)y' - 1$, $y(0) = 0$, $y'(0) = 1$, $n = 4$.
10. $y'' = x^2 y + e^x$, $y(0) = y'(0) = 1$, $n = 4$.
11. $y' = \cos x - y^2 + x$, $y(0) = 1$, $n = 3$.
12. $y'' + y \sin x - y' = 0$, $y(0) = 1$, $y'(0) = 0$, $n = 3$.
13. $y' = y^2 + \sin^2 x$, $y(0) = 2$, $n = 4$.
14. $y' = \sin 4x + \cos 2y$, $y(0) = 0$, $n = 3$.
15. $y' = \cos y + x e^y$, $y(0) = 0$, $n = 2$.
16. $y' = xy + e^y - 5x$, $y(0) = 0$, $n = 3$.
17. $y'' = xy' - \sin y + e^x$, $y(0) = 0$, $y'(0) = 1$, $n = 3$.

18. $y' = \cos x + x^2 + y^2$, $y(0) = 1$, $n = 4$.
 19. $y'' = xy' - y + \sin x$, $y(0) = y'(0) = 0$, $n = 2$.
 20. $y' = y^3 - \sin x$, $y(0) = 1$, $n = 4$.
 21. $y' = xy^2 - \cos x$, $y(0) = 1$, $n = 3$.
 22. $y' = ye^{-x} + \ln y$, $y(0) = 1$, $n = 3$.
 23. $y'' = (1 + x^2)y + e^x$, $y(0) = -2$, $y'(0) = 2$, $n = 4$.
 24. $y' = \cos x + e^y + xy$, $y(0) = 0$, $n = 3$.
 25. $y' = \ln(x+1) + xe^y$, $y(0) = 0$, $n = 2$.

Задача 4 (2 балла)

Вычислите интеграл с точностью до 0,001, взяв необходимое число слагаемых.

$$1. \int_0^{0,1} e^{-6x^2} dx.$$

$$2. \int_0^1 \cos x^2 dx.$$

$$3. \int_0^{0,1} \frac{1 - e^{-2x}}{x} dx.$$

$$4. \int_0^1 \frac{\ln\left(1 + \frac{x}{5}\right)}{x} dx.$$

$$5. \int_0^1 \frac{x - \sin x}{x^3} dx.$$

$$6. \int_0^{0,2} e^{-3x^2} dx.$$

$$7. \int_0^{0,2} \sin(25x^2) dx.$$

$$8. \int_0^{0,5} \cos(4x^2) dx.$$

$$9. \int_0^1 x^2 e^{-x^2} dx.$$

$$10. \int_0^{0,4} \frac{\ln\left(1 + \frac{x}{2}\right)}{x} dx.$$

$$11. \int_0^1 \frac{1 - \cos x}{x} dx.$$

$$12. \int_0^{0,3} e^{-2x^2} dx.$$

$$13. \int_0^{0,4} \sin\left(\frac{5x}{2}\right)^2 dx.$$

$$14. \int_0^{0,2} \cos(25x^2) dx.$$

$$15. \int_0^1 \frac{\sin x}{x} dx.$$

$$16. \int_0^{0,4} \frac{1 - e^{-\frac{x}{2}}}{x} dx.$$

$$17. \int_0^{0,1} \frac{\ln(1 + 2x)}{x} dx.$$

$$18. \int_0^{0,4} e^{-\frac{3x^2}{4}} dx.$$

$$19. \int_0^{0,5} \sin(4x^2) dx.$$

$$20. \int_0^{0,4} \cos\left(\frac{5x}{2}\right)^2 dx.$$

$$21. \int_0^{0,25} \frac{\sqrt{x} dx}{e^x}.$$

$$22. \int_0^{0,5} \frac{\operatorname{arctg} x}{x} dx.$$

$$23. \int_0^{0,5} e^{-\frac{3x^2}{25}} dx.$$

$$24. \int_0^1 \sin(x^2) dx.$$

$$25. \int_0^{0,1} \cos(100x^2) dx.$$